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STATE OF THERMAL STRESS AND STRAIN OF A PLATE WEAKENED BY A RECTANGULAR HOLE*

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By using the method of continuation of functions a solution is obtained for the stationary heat conduction problem and for the corresponding static problem of thermo-elasticity for an infinite plate weakened by a rectangular hole.

1. Solution of the heat conduction problem. Let us consider a homogeneous isotropic unbounded plate of thickness 2δ with a rectangular cutout $|x_i| < a_i$ $(i = 1, 2)$. Heat transfer from the external medium occurs by Newton's law through the surface of the cutout and the side surfaces $x_3 = \pm\delta$. We ensure the temperature of the medium flowing over the surfaces $x_3 = \pm\delta$ to be zero, while the temperature of the medium flowing over the plate rectangular boundary is t_c. We then have the third boundary value problem for the Helmholtz equation in the domain external to the rectangle /1/ to determine the stationary temperature field T in the plate. We use the method of continuation of functions /2/ to solve this problem. To do this we introduce a new unknown function Θ that agrees with the desired function of the temperature T outside the rectangle and equals zero within, i.e.,

$$\Theta = TM(x_1, x_2) \tag{1.1}$$
$$M(x_1, x_2) = 1 - M(x_1)M(x_2), \quad M(x_i) = S_+(x_i + a_i) - S_-(x_i - a_i)$$
$$S_\pm(\xi) = \begin{cases} 1, & \xi > 0 \\ 0{,}5 \mp 0{,}5, & \xi = 0 \\ 0, & \xi < 0 \end{cases}$$

Taking account of the symmetry of the problem relative to the coordinate axes and the boundary conditions on the rectangle contour, we obtain an equation with singular coefficients for the function

$$\frac{\partial^2\Theta}{\partial x_1^2} + \frac{\partial^2\Theta}{\partial x_2^2} - \varkappa^2\Theta = \sum_{i=1}^{2}\{h_i(T|_{x_i=a_i} - t_c)M(x_{i\pm1}) \times [\delta_+(x_i + a_i) + \delta_-(x_i - a_i)] - T|_{x_i=a_i}M(x_{i\pm1}) \times [\delta_+'(x_i + a_i) - \delta_-'(x_i - a_i)]\} \tag{1.2}$$
$$h_i = \frac{\alpha_i}{\lambda}, \quad \varkappa^2 = \frac{\alpha_3}{\lambda\delta}, \quad i \pm 1 = \begin{cases} 2, & i = 1 \\ 1, & i = 2 \end{cases}$$

(λ is the thermal conductivity, α_3 and α_i $(i = 1, 2)$ are heat transfer coefficients from the surfaces $x_3 = \pm\delta$, and $|x_i| < a_i$, $|x_{i\pm1}| = a_{i\pm1}$).

The values of the function T on the rectangle contour that are in (1.2) are expanded in a Fourier series

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$$T\,|_{x_i=a_i} M(x_{i\pm1}) = \sum_{n=0}^{\infty} c_n^{(i)} \cos\lambda_n^{(i\pm1)} x_{i\pm1} M(x_{i\pm1}) \tag{1.3}$$

$$c_n^{(i)} = \frac{\varepsilon(n)}{a_{i\pm1}} \int_{-a_{i\pm1}}^{a_{i\pm1}} T\,|_{x_i=a_i} \cos\lambda_n^{(i\pm1)} x_{i\pm1}\, dx_{i\pm1} \tag{1.4}$$

$$\lambda_n^{(i)} = \frac{\pi n}{a_i}, \quad \varepsilon(n) = \begin{cases} 0{,}5, & n=0 \\ 1, & n=1,2,\ldots \end{cases}$$

In order to ensure continuity of the solution at an angular point, we assume that the Fourier coefficients $c_n^{(i)}$ satisfy the relationship

$$\sum_{n=0}^{\infty} (-1)^n c_n^{(1)} = \sum_{n=0}^{\infty} (-1)^n c_n^{(2)} \tag{1.5}$$

Substituting expansion (1.3) into (1.2) and then using the Fourier integral transform in the coordinates, we obtain a solution of (1.2)

$$\Theta = \frac{1}{\pi} \sum_{i=1}^{2} \int_0^{\infty} \Big\{ \sum_{n=0}^{\infty} (-1)^n c_n^{(i)} p_{c_x s_a}(\eta, x_{i\pm1}, \lambda_n^{(i\pm1)}) \Big[\frac{h_i}{\gamma} \psi_1^+(\gamma, x_i) + \tag{1.6}$$

$$\psi_2^-(\gamma, x_i) \Big] + t_c h_i \frac{\eta}{\gamma} p_{c_x s_a}(\eta, x_{i\pm1}, 0)\, \psi_1^+(\gamma, x_i) \Big\} d\eta$$

$$\psi_1^{\pm}(\eta, x_i) = \exp(-|x_i + a_i|_+ \eta) \pm \exp(-|x_i - a_i|_- \eta)$$

$$\psi_2^{\pm}(\eta, x_i) = \exp(-|x_i + a_i|_+ \eta)\, \mathrm{sign}_{\pm}(x_i + a_i) \pm$$

$$\exp(-|x_i - a_i|_- \eta)\, \mathrm{sign}_-(x_i - a_i)$$

$$p_{c_x s_a}(\eta, x_i, \lambda_n^{(i)}) = \frac{\cos\eta x_i \sin\eta a_i}{\eta^2 - (\lambda_n^{(i)})^2}$$

$$\mathrm{sign}_{\pm} x = 2S_{\pm}(x) - 1, \quad |x|_{\pm} = x\, \mathrm{sign}_{\pm} x, \quad \gamma = \sqrt{\varkappa^2 + \eta^2}$$

The unknown Fourier coefficients $c_n^{(i)}$ in the solution (1.6) are found from the following system of linear algebraic equations by using (1.4):

$$c_k^{(i)} + \sum_{n=0}^{\infty} A_{kn}^{(i,i)} c_n^{(i)} = D_k^{(i)} \quad (i=1,2;\ k=0,1,\ldots) \tag{1.7}$$

$$D_k^{(i)} = \sum_{n=0}^{\infty} A_{kn}^{(i,i\pm1)} c_n^{(i\pm1)} + B_k^{(i)}, \quad f_{1,i}^{\pm}(\eta) = 1 \pm \exp(-2a_i\eta)$$

$$A_{kn}^{(i,i)} = \frac{4\varepsilon(k)}{a_{i+1}} \int_0^{\infty} g_{ss}^-(\lambda_n^{(i\pm1)}, \lambda_k^{(i\pm1)}, \eta) \Big[\exp(-2a_i\gamma) + \frac{h_i}{\gamma} f_{1,i}^+(\gamma) \Big] d\eta$$

$$A_{kn}^{(i,i\pm1)} = (-1)^{n+k+1} \frac{4\varepsilon(k)}{\pi a_{i\pm1}} \int_0^{\infty} \frac{\eta(h_{i\pm1} + \gamma)}{\gamma^2 + (\lambda_k^{(i\pm1)})^2} p_{c_a s_a}(\eta, a_i, \lambda_n^{(i)}) f_{1,i\pm1}^-(\gamma)\, d\eta$$

$$B_k^{(i)} = 2t_c \frac{\varepsilon(k)}{a_{i\pm1}} \int_0^{\infty} \Big\{ \frac{2h_i\eta^2}{\gamma} g_{ss}^-(0, \lambda_k^{(i+1)}, \eta) f_{1,i}^+(\eta) + \frac{(-1)^k h_{i\pm1}\eta}{\pi[\gamma^2 + (\lambda_k^{(i\pm1)})^2]} \times$$

$$\sin 2a_i\eta f_{1,i\pm1}^+(\gamma) \Big\} d\eta, \quad g_{ss}^{\pm}(\lambda_n^{(i)}, \lambda_k^{(i)}, \eta) = \frac{(-1)^{n+k} \sin^2\eta a_i}{\pi(\eta^2 - (\lambda_n^{(i)})^2)(\eta^2 \pm (\lambda_k^{(i)})^2)}$$

The coefficients $A_{kn}^{(i,i)}$ of system (1.7) equal the scalar product of elements of a linearly independent system of functions in the space $L_2(0, \infty)$

$$\frac{2(-1)^n}{\sqrt{\pi a_{i\pm1}}} \frac{\eta \sin\eta a_{i\pm1}}{\eta^2 - (\lambda_n^{(i\pm1)})^2} \Big[\exp(-2a_i\gamma) + \frac{h_i}{\gamma} f_{1,i}^+(\gamma) \Big]^{1/2} \quad (n=0,1,\ldots) \tag{1.8}$$

Therefore, the third boundary value problem for the Helmholtz equation in an external domain to a rectangle is reduced to the solution of an infinite system of linear algebraic equations.

2. Foundation of the method of reduction of the solution of system (1.7).

We shall seek the solution of system (1.7) that is convergent in the norm of the space l^2, i.e.,

$$\sum_{k=0}^{\infty} [c_k^{(i)}]^2 < \infty \quad (i=1,2) \tag{2.1}$$

The estimates /3, 4/

$$\sum_{k,n=0}^{\infty} [A_{kn}^{(i,i)}]^2 < \infty, \quad \sum_{k=0}^{\infty} [D_k^{(i)}]^2 < \infty \quad (i = 1, 2) \tag{2.2}$$

hold for the coefficients of system (1.7).

We consider the first $2m$ equations of system (1.7) with $2m$ unknowns

$$c_k^{(i)} + \sum_{n=0}^{m-1} A_{kn}^{(i,i)} c_n^{(i)} = D_k'^{(i)} \quad (i = 1, 2;\ k = 0, 1, \ldots, m-1) \tag{2.3}$$

$$D_k'^{(i)} = \sum_{n=0}^{m-1} A_{kn}^{(i,i\pm1)} c_n^{(i\pm1)} + B_k^{(i)}$$

System (2.3) has a unique solution.

Indeed, since the coefficients $A_{kn}^{(i,i)}$ equal the scalar product of elements of a linearly independent system of functions, the determinant of system (2.3) can be represented as the sum of one and Gramm determinants from the first to m-th order. Since the system of functions (1.8) is linearly independent, the determinant is positive for arbitrary m.

If the equations of system (2.3) for $k = 0$ are multiplied by 2, the matrix of the coefficients for the $c_n^{(i)}$ of this system become symmetric. In addition it is positive-definite.

It follows from the above that the theory of the solvability of infinite systems /5/ is applicable to system (1.7), from which this assertion follows: system (1.7) has a unique solution satisfying condition (2.1). The approximate solution of system (1.7) can be found by the method of reduction.

3. Solution of the thermo-elasticity problem. We will determine the temperature stresses in a plate due to the temperature field (1.6). The equilibrium equations in displacements for an isotropic plate have the form /1/

$$\frac{1-\nu}{1+\nu} \Delta u_i + \frac{\partial}{\partial x_i} \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_{i\pm1}}{\partial x_{i\pm1}} \right) = 2\alpha_t \frac{\partial T}{\partial x_i} \quad (i = 1, 2) \tag{3.1}$$

where ν is Poisson's ratio, α_t is the temperature coefficient of linear expansion, and Δ is the Laplace operator.

We assume that the plate is free of external load, i.e.,

$$\sigma_{ii}|_{x_i=\pm a_i} = \sigma_{12}|_{x_i=\pm a_i} = 0, \quad |x_{i\pm1}| < a_{i\pm1} \tag{3.2}$$

The stress tensor components σ_{ij} are connected with the displacement vector components (u_1, u_2) by the Duhamel-Neumann relationships /1/.

As in solving heat conduction problems we similarly introduce new unknown functions

$$U_i = u_i M(x_1, x_2), \quad \Omega_{ij} = \sigma_{ij} M(x_1, x_2) \quad (i, j = 1, 2) \tag{3.3}$$

Taking into account the symmetry of the problem relative to the coordinate axes and the boundary conditions on the contour of the rectangle (3.2), we obtain a system of differential equations containing delta-functions and their derivatives for U_i

$$\begin{aligned}
&\frac{2}{1+\nu}\frac{\partial^2 U_i}{\partial x_i^2} + \frac{1-\nu}{1+\nu}\frac{\partial^2 U_i}{\partial x_{i\pm1}^2} + \frac{\partial^2 U_{i\pm1}}{\partial x_1 \partial x_2} = 2\alpha_t \Theta - \\
&\frac{1-\nu}{1+\nu}\frac{\partial u_{i\pm1}}{\partial x_{i\pm1}}\bigg|_{x_i=a_i} M(x_{i\pm1})[\delta_+(x_i+a_i) - \delta_-(x_i-a_i)] + \\
&\frac{2}{1+\nu}\frac{\partial u_{i\pm1}}{\partial x_i}\bigg|_{x_{i\pm1}=a_{i\pm1}} M(x_i)[\delta_+(x_{i\pm1}+a_{i\pm1}) + \delta_-(x_{i\pm1}-a_{i\pm1})] + \\
&\frac{2}{1-\nu} u_i|_{x_i=a_i} M(x_{i\pm1})[\delta_+'(x_i+a_i) + \delta_-'(x_i-a_i)] - \\
&\frac{1-\nu}{1+\nu} u_i|_{x_{i\pm1}=a_{i\pm1}} M(x_i)[\delta_+'(x_{i+1}+a_{i+1}) - \delta_-'(x_{i\pm1}-a_i)] + \\
&u_{i\pm1}|_{x_1=a_1, x_2=a_2} [\delta_+(x_{i\pm1}+a_{i\pm1}) + \delta_-(x_{i\pm1}-a_{i\pm1})] \times \\
&[\delta_+(x_i+a_i) - \delta_-(x_i-a_i)] \quad (i = 1, 2)
\end{aligned} \tag{3.4}$$

The values of the functions u_i on the plate rectangular boundaries that are in system (3.4) are expanded in a Fourier series

$$u_i|_{x_i=a_i} = \sum_{n=0}^{\infty} r_n^{(i)} \cos \lambda_n^{(i\pm1)} x_{i\pm1}, \quad \lambda_n^{(i\pm1)} = \frac{\pi n}{a_{i\pm1}}$$

$$u_i\,|_{x_{i\pm1}=a_{i\pm1}} = \sum_{n=0}^{\infty} t_n^{(i)} \cos \mu_n^{(i)} x_i, \qquad \mu_n^{(i)} = \frac{\pi(n+0{,}5)}{a_i}$$

Substituting the expansions into system (3.4), and using the Fourier transform in the coordinates, we obtain the following expressions for the functions $\Omega_{ij}\,(i, j = 1, 2$, and E is the elastic modulus)

$$\begin{aligned}
\Omega_{ii} = {} & \frac{\alpha_t E}{\pi\varkappa^2}\Big\langle \sum_{n=0}^{\infty} (-1)^n c_n^{(i)} \int_0^{\infty} p_{c_x s_a}(\eta, x_{i\pm1}, \lambda_n^{(i\pm1)})\,\eta^2 \times \\
& \Big\{\eta\,[\psi_2^-(\eta, x_i) - \psi_2^-(\gamma, x_i)] + h_i\Big[\psi_1^+(\eta, x_i) - \frac{\eta}{\gamma}\psi_1^+(\gamma, x_i)\Big]\Big\}\,d\eta - \\
& \sum_{n=0}^{\infty} (-1)^n c_n^{(i\pm1)} \int_0^{\infty} \eta p_{c_x s_a}(\eta, x_i, \lambda_n^{(i)})\,\{\eta^2\psi_2^-(\eta, x_{i\pm1}) - \\
& \gamma^2\psi_2^-(\gamma, x_{i\pm1}) + h_{i\pm1}\,[\eta\psi_1^+(\eta, x_{i\pm1}) - \gamma\psi_1^+(\gamma, x_{i\pm1})]\}\,d\eta + \\
& t_c \int_0^{\infty} \Big\{h_i\eta^2 p_{c_x s_a}(\eta, x_{i\pm1}, 0)\Big[\frac{\eta}{\gamma}\psi_1^+(\gamma, x_i) - \psi_i^+(\eta, x_i)\Big] + \\
& h_{i\pm1}\eta p_{c_x s_a}(\eta, x_i, 0)\,[\eta\psi_1^+(\eta, x_{i\pm1}) - \gamma\psi_1^+(\gamma, x_{i\pm1})]\Big\}\,d\eta\Big\rangle + \\
& \frac{E}{2\pi}\Big\{\sum_{n=0}^{\infty} (-1)^n r_n^{(i\pm1)}\,[\Phi_{33n}^+(c_x s_a, x_i, \lambda_n^{(i)}, x_{i\pm1}) - \\
& \Phi_{21n}^+(c_x s_a, x_i, \lambda_n^{(i)}, x_{i\pm1})] + \sum_{n=0}^{\infty} (-1)^n r_n^{(i)}\,[\Phi_{33n}^+(c_x s_a, x_{i\pm1}, \lambda_n^{(i\pm1)}, x_i) + \\
& \Phi_{21n}^+(c_x s_a, x_{i\pm1}, \lambda_n^{(i\pm1)}, x_i)] + \sum_{n=0}^{\infty} (-1)^n t_n^{(i)}\,[2\Phi_{22n}^-(c_x s_a, x_i, \mu_n^{(i)}, x_{i\pm1}) - \\
& \Phi_{34n}^-(c_x s_a, x_i, \mu_n^{(i)}, x_{i\pm1})] + \sum_{n=0}^{\infty} (-1)^n t_n^{(i\pm1)}\Phi_{31n}^-(c_x s_a, x_{i\pm1}, \mu_n^{(i\pm1)}, x_i)\Big\}
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\Omega_{12} = {} & \frac{E}{2\pi}\sum_{i=1}^{2}\sum_{n=0}^{\infty} (-1)^n \Big\langle 2\,\frac{\alpha_t}{\varkappa^2}\Big\{c_n^{(i)} \int_0^{\infty} p_{s_x s_a}(\eta, x_{i\pm1}, \lambda_n^{(i\pm1)})\,\eta^2 \times \\
& [\eta\psi_1^-(\eta, x_i) - \gamma\psi_1^-(\gamma, x_i) - h_i(\psi_2^-(\eta, x_i) - \psi_2^-(\gamma, x_i))]\,d\eta\Big\} - \\
& r_n^{(i)}\Phi_{34n}^+(s_x s_a, x_{i\pm1}, \lambda_n^{(i\pm1)}, x_i) + t_n^{(i\pm1)}\,[\Phi_{33n}^-(s_x s_a, x_{i\pm1}, \mu_n^{(i\pm1)}, x_i) - \\
& 2\Phi_{21n}^-(s_x c_a, x_{i\pm1}, \mu_n^{(i\pm1)}, x_i)]\Big\rangle
\end{aligned}$$

$$p_{s_x s_a}(\eta, x_i, \lambda_n^{(i)}) = \frac{\sin\eta a_i \sin\eta x_i}{\eta^2 - (\lambda_n^{(i)})^2}$$

$$\Phi_{lmn}^{\pm}(s_x s_a, x_i, \lambda_n^{(i)}, x_{i\pm1}) = \int_0^{\infty} \eta^2 p_{s_x s_a}(\eta, x_i, \lambda_n^{(i)})\,\psi_m^{\pm}(\eta, x_{i\pm1})\,d\eta$$

$$\begin{aligned}
\psi_3^{\pm}(\eta, x_i) &= |x_i + a_i|_+ \exp(-|x_i + a_i|_+\eta) \pm \\
&\quad |x_i - a_i|_- \exp(-|x_i - a_i|_-\eta) \\
\psi_4^{\pm}(\eta, x_i) &= (x_i + a_i)\exp(-|x_i + a_i|_+\eta) \pm \\
&\quad (x_i - a_i)\exp(-|x_i - a_i|_-\eta)
\end{aligned}$$

We find the Fourier coefficients $t_n^{(i)}$ and $r_n^{(i)}$ in the solution of the thermo-elasticity problem from the infinite system of linear algebraic equations

$$\begin{aligned}
& r_k^{(i)} + \sum_{n=0}^{\infty} R_{kn}^{(i,\,i)} r_n^{(i)} = \sum_{n=0}^{\infty} (R_{kn}^{(i,\,i\pm1)} r_n^{(i\pm1)} + N_{kn}^{(i,\,i)} t_n^{(i)} + \\
& \quad N_{kn}^{(i,\,i\pm1)} t_n^{(i\pm1)}) + F_k^{(i)} \\
& t_k^{(i)} + \sum_{n=0}^{\infty} G_{kn}^{(i,\,i)} t_n^{(i)} = \sum_{n=0}^{\infty} (G_{kn}^{(i,\,i\pm1)} t_n^{(i\pm1)} + T_{kn}^{(i,\,i)} r_n^{(i)} + \\
& \quad T_{kn}^{(i,\,i\pm1)} r_n^{(i\pm1)} + V_k^{(i)} \\
& F_k^{(i)} = \sum_{n=0}^{\infty} (D_{kn}^{(i,\,i)} c_n^{(i)} + D_{kn}^{(i,\,i\pm1)} c_n^{(i\pm1)}) + H_k^{(i)} \\
& V_k^{(i)} = \sum_{n=0}^{\infty} (Q_{kn}^{(i,\,i)} c_n^{(i)} + Q_{kn}^{(i,\,i\pm1)}) + Z_k^{(i)}
\end{aligned} \tag{3.6}$$

$$D_{kn}^{(i,\,i)} = \frac{2E\alpha_t}{a_{i\pm1}\varkappa^2}\,\varepsilon(k)\int\limits_0^\infty \eta^2 g_{ss}^-(\lambda_n^{(i\pm1)}, \lambda_k^{(i\pm1)}, \eta)\,[\eta f_{1,\,i}^-(\eta) - \gamma f_{1,\,i}^-(\gamma) + h_i f_{2,\,i}(\eta)]\,d\eta$$

$$D_{kn}^{(i,\,i\pm1)} = -\frac{2E\alpha_t}{a_{i\pm1}\varkappa^2}\,\varepsilon(k)\int\limits_0^\infty \eta^2 g_{ss}^-(0, \lambda_n^{(i)}, \eta)\,[(\eta + h_{i\pm1}) f_{3,\,i\pm1}(\eta, \lambda_k^{(i\pm1)}) -$$
$$(h_{i\pm1} + \gamma) f_{3,\,i\pm1}(\gamma, \lambda_k^{(i\pm1)})]\,d\eta$$

$$R_{kn}^{(i,\,i)} = 4\,\frac{\varepsilon(k)}{a_{i\pm1}}\int\limits_0^\infty g_{ss}^-(\lambda_n^{(i\pm1)}, \lambda_k^{(i\pm1)}, \eta)\,\eta^2\,[(1+\nu)a_i\eta + 1] \times \exp(-2a_i\eta)\,d\eta$$

$$R_{kn}^{(i,\,i\pm1)} = 2\,\frac{\varepsilon(k)}{a_{i\pm1}}\int\limits_0^\infty g_{ss}^+(\lambda_n^{(i)}, \lambda_k^{(i\pm1)}, \eta)\,\eta^2\,[(1+\nu) f_4(\eta, \lambda_k^{(i\pm1)}) - (1-\nu) f_{1,\,i\pm1}^-(\eta)]\,d\eta$$

$$N_{kn}^{(i,\,i)} = 2\,\frac{\varepsilon(k)}{a_{i\pm1}}\int\limits_0^\infty \eta^2 g_{sc}^+(\mu_n^{(i)}, \lambda_k^{(i\pm1)}, \eta)\,\{2 f_{1,\,i\pm1}^-(\eta) +$$
$$(1+\nu) f_4(\eta, \lambda_k^{(i\pm1)})\}\,d\eta$$

$$N_{kn}^{(i,\,i\pm1)} = 2\,\frac{\varepsilon(k)}{a_{i\pm1}}\int\limits_0^\infty \eta^2 g_{sc}^-(\mu_n^{(i\pm1)}, \lambda_k^{(i\pm1)}, \eta)\,[(1-\nu) f_{1i}^-(\eta) -$$
$$2(1+\nu)a_i\eta\exp(-2a_i\eta)]\,d\eta$$

$$H_k^{(i)} = \frac{2E\alpha_t}{a_{i\pm1}}\,\varepsilon(k)\,t_c\int\limits_0^\infty \{h_i g_{ss}^-(\lambda_k^{(i\pm1)}, 0, \eta) f_{2i}(\eta) +$$
$$(-1)^k \pi^{-1} h_{i\pm1}\sin^2\eta a_{i\pm1}\,[f_3(\eta, \lambda_k^{(i\pm1)}) - f_3(\gamma, \lambda_k^{(i\pm1)})]\}\,d\eta$$

$$Q_{kn}^{(i,\,i)} = \frac{2E\alpha_t}{a_i\varkappa^2}\,(-1)^k\int\limits_0^\infty \eta g_{sc}^-(\lambda_n^{(i\pm1)}, 0, \eta)\,[(\eta + h_i)\eta f_3(\eta, \mu_n^{(i)}) + (\gamma + h_i)\gamma f_3(\gamma, \mu_n^{(i)})]\,d\eta$$

$$Q_{kn}^{(i,\,i\pm1)} = \frac{2E\alpha_t}{a_i\varkappa^2}\int\limits_0^\infty \eta^3 g_{sc}^-(\lambda_n^{(i)}, \mu_k^{(i)}, \eta)\left[f_{2,\,i\pm1}(\eta) + h_{i\pm1}\left(\frac{f_{1,\,i}^+(\eta)}{\eta} - \frac{f_{1,\,i}^+(\gamma)}{\gamma}\right)\right]d\eta$$

$$T_{kn}^{(i,\,i)} = -\frac{2}{a_i}\int\limits_0^\infty \eta^2 g_{sc}^+(\lambda_n^{(i\pm1)}, \mu_k^{(i)}, \eta)\,[2 f_{1,\,i}^-(\eta) + (1+\nu) f_4(\eta, \mu_k^{(i)})]\,d\eta$$

$$T_{kn}^{(i,\,i\pm1)} = -\frac{4}{a_i}\int\limits_0^\infty \eta^2 g_{sc}^-(\lambda_n^{(i)}, \mu_k^{(i)}, \eta)\,[2(1+\nu)a_{i\pm1}\eta -$$
$$(1+\nu) f_{1,\,i\pm1}^+(\eta)]\,d\eta$$

$$G_{kn}^{(i,\,i)} = \frac{4}{a_i}\int\limits_0^\infty \eta^2 g_{ss}^-(\mu_n^{(i)}, \mu_k^{(i)}, \eta)\,(1 + (1+\nu)a_{i\pm1}\eta)\exp(-2a_{i\pm1}\eta)\,d\eta$$

$$G_{kn}^{(i,\,i\pm1)} = \frac{2}{a_i}\int\limits_0^\infty \eta^2 g_{cc}^+(\mu_n^{(i\pm1)}, \mu_k^{(i)}, \eta)\,[(1-\nu) f_{1,\,i}^-(\eta) +$$
$$(1+\nu) f_4(\eta, \mu_k^{(i)})]\,d\eta$$

$$Z_k^{(i)} = -\frac{2}{a_i}\,t_c\int\limits_0^\infty \left\{(-1)^k h_i\,\frac{\sin 2\eta a_{i\pm1}}{\eta}\,[\eta f_3(\eta, \mu_k^{(i)}) - \gamma f_3(\gamma, \mu_k^{(i)})] +\right.$$
$$\left.2h_{i\pm1} g_{cs}^-(0, \mu_k^{(i)}, \eta)\left[f_{1,\,i\pm1}^+(\eta) - \frac{\eta}{\gamma} f_{1,\,i\pm1}^+(\gamma)\right]\right\}d\eta$$

$$f_{2,\,i}(\eta) = \exp(-2a_i\eta) - \exp(-2a_i\gamma),\quad f_3(\eta, \lambda_k^{(i)}) = \frac{f_{1,\,i}^-(\eta)}{\eta^2 + (\lambda_k^{(i)})^2}$$

$$f_4(\eta, \lambda_k^{(i)}) = \frac{\eta^2 - (\lambda_k^{(i)})^2}{\eta^2 + (\lambda_k^{(i)})^2} f_{1,\,i}^-(\eta) - 2a_i\eta\exp(-2a_i\eta)$$

Estimates analogous to the estimates (2.2), obtained for system (1.7), hold for coefficients of system (3.6). System (3.6) has a solution convergent in the norm of the space l^2. The approximate solution can be obtained by the method of reduction.

4. Behaviour of the solution of an angular point. We note that the solution of the thermo-elasticity problem (3.5) can be obtained in a different form if the convolution theorem for the Fourier transform is used. For instance, the integral

$$\int\limits_0^\infty \frac{\eta^2\sin\eta a_1\cos\eta x_1}{\eta^2 - (\lambda_n^{(1)})^2}\,(\eta\,|x_2 - a_2|_- - 1)\exp(-|x_2 - a_2|_-\eta)\,d\eta$$

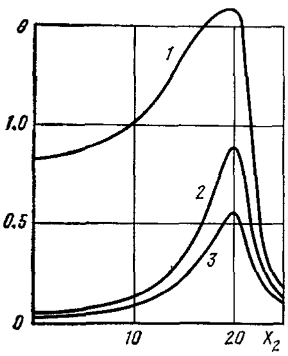
is written, apart from a constant factor, in the form

$$\int_{-a_1}^{a_1} \cos \lambda_n^{(1)} x \, \frac{6(x-x_1)^2(x_2-a_2)^2-(x_2-a_2)^4-(x-x_1)^4}{((x-x_1)^2+(x_2-a_2)^2)^3}\, dx \tag{4.1}$$

Integrating (4.1) twice by parts, a power-law singularity can be extracted that occurs at the angular point (a_1, a_2) in the form

$$\frac{(x_1-a_1)\,[(x_1-a_1)^2-(x_2-a_2)^2]}{[(x_1-a_1)^2+(x_2-a_2)^2]^2}$$

Analogous power-law singularities are obtained at angular points when investigating the other components in the expression for Ω_{ij}.



5. Results of numerical investigations. Formally setting $\alpha_1 t_c = \alpha_2 t_c = q_0$, $\alpha_1 = \alpha_2 = 0$ in (1.6) and (3.5), we will obtain the solution of the stationary heat conduction problem and the corresponding static thermo-elasticity problem for a plate with a rectangular cutout on whose boundaries the heat flux q_0 is given. For this case the dimensionless temperature field $\theta = T\lambda/q_0\delta$ was computed as a function of $X_2 = x_2/\delta$ for $A_1 = a_1/\delta = 10$, $A_2 = a_2/\delta = 20$, $\mathrm{Bi} = \alpha_3\delta/\lambda = 0.1$ and different $X_1 = x_1/\delta$. A 20×20 and 40×40 matrix of the truncated system was formed in solving system (1.7) by the method of reduction. Results of the calculations are practically identical. The results of the temperature field computations are represented as graphs in the figure for $X_1 = 10$; 11.25, 12.5 (curves *1, 2, 3,* respectively). It follows from the graphs that the maximum value of the temperature is achieved at the angular point. The temperature is equalized with distance from the boundary.

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ACTION OF A UNIFORMLY VARIABLE MOVING FORCE ON A TIMOSHENKO BEAM ON AN ELASTIC FOUNDATION. TRANSITIONS THROUGH THE CRITICAL VELOCITIES*

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The vibrations of an infinite Timoshenko-type beam on an elastic foundation subjected to a force whose point of application moves over the beam with constant acceleration are considered. Resonance effects associated with the transition of the velocity of motion of the load through three critical values characteristic for the system being considered are studied. Asymptotic representations are constructed for the solution of the problem corresponding to the load acceleration approaching zero.

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